Lecture 5

Material and Homework

Read:

Chapter 5 Sections Yariv and Yeh

Chapter 6 Sections 6.1-6.5 Yariv and Yeh

Appendices 1 and 2 in Coldren (on reserve)

Laser Oscillation Frequency

- We saw that for the FP cavity, the phase condition that must be met for the cavity to be "resonant" is satisfied for an infinite number of wavelengths (frequencies)

$$2(k + \Delta k(\omega))l = 2m\pi, \text{ m=1,2,3...}$$

- However, the gain medium, inside the FP cavity (or any laser cavity) has limited bandwidth. So the laser will oscillate at all frequencies that satisfy both the phase condition and the gain condition

$$r_1 r_2 e^{(\gamma(\omega)-\alpha)l} = 1$$

- Defining the m[th] frequency in a passive cavity (N_2-N_1=0) by $\nu_m = mc/2nl$, we can re-write the phase condition as

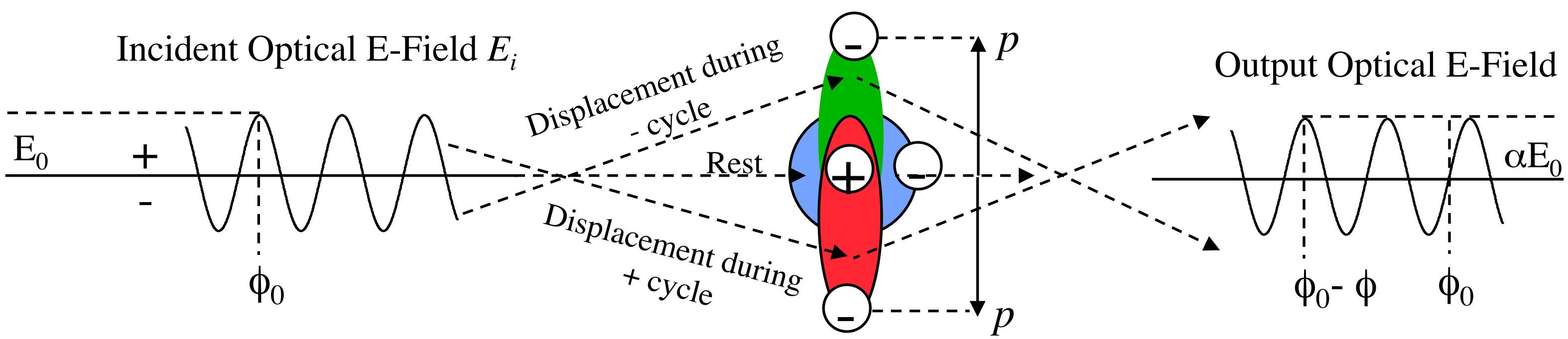
$$kl\left(1 + \frac{\chi'(\nu)}{2n^2}\right) = m\pi$$

- Now we need to discuss in more detail the origin of

$$\chi'(\nu) \text{ and } \chi''(\nu)$$

Radiation and Atomic Systems

- Chapter 5, Yariv
- In a dielectric material, an incident optical E-field oscillating at frequency w will induce physical displacement of a bound electron from its rest state at the same oscillation frequency. We call the the ***induced dipole moment or displacement***



α is field attenuation

p is the induced dipole moment

α_p is the atomic polarizability

ϕ_0 is the input field phase reference

ϕ is the output field phase

$$\mathbf{p} = \alpha_p \mathbf{E}_i$$

Dielectric Constant and Index of Refraction

- For a homogeneous material of N atoms per unit volume, the polarization is defined by the contribution of all atoms in that volume that interact with the optical E-field

$$\mathbf{P} \approx N\mathbf{p} = N\alpha_p \mathbf{E_i} \equiv \varepsilon_0 \chi_e \mathbf{E}_i$$

- Where we have defined the ***vacuum permittivity*** ε_0 and ***material electric susceptibility*** χ_ε by

$$N\alpha_p \equiv \varepsilon_0 \chi_e$$

- Using the constitutive relation $\mathbf{D}=\varepsilon\mathbf{E}=\varepsilon_0\mathbf{E}+\mathbf{P}$**,** the ***dielectric constant*** is defined as

$$\varepsilon = \varepsilon_0 + \frac{\mathbf{P}}{\mathbf{E}} = \varepsilon_0 + \varepsilon_0\chi_e = \varepsilon_0\left(1+\chi_e\right)$$

$$= \varepsilon_0\left(1 + {N\alpha_p}/{\varepsilon_0}\right)$$

- And we can now define for a non-magnetic medium (which dielectrics are at optical frequencies), the ***optical index of refraction n***

$$n^2 = 1 + \chi_e = 1 + {N\alpha_p}/{\varepsilon_0}$$

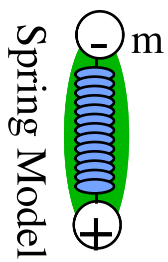
$$n = \sqrt{1+\chi_e} = \sqrt{1 + {N\alpha_p}/{\varepsilon_0}}$$

Classical Electron Model

- Now we need a basic model that describes the motion, or displacement of the electron (dipole) in the presence of an external force (incident optical field)
- Consider the classical equation of motion that describes displacement (position X) of an electron with charge *q* and mass *m* attached to an atom (as if connected by a spring) with a dampening coefficient (loss) γ and resonant frequency ω_0 in response to an applied field

$$m\frac{d}{dt^2}X + m\gamma\frac{d}{dt}X + m\omega_0^2 X = -qE_a$$

- Lets assume the atom can be described as a two level system with energy levels E_1 and E_2 such that the atomic resonance is $\omega_0 = (E_2 - E_1)/\hbar$ and the applied electric field is of the form $E = E_0 e^{i\omega t}$. Then the eletron position (relative to rest) and induced dipole moment of the atom can be described as



$$X = \frac{-qE_0}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}e^{i\omega t}$$

$$p = -qX = \frac{-q^2 E_0}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}e^{i\omega t}$$

Classical Electron Model

- The dielectric constant can now be written in terms of the spring model

$$p = \alpha_p E_a$$

$$\alpha_p = \frac{q^2}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

$$n^2 = 1 + \chi = 1 + {}^{N\alpha_p}\!/_{\varepsilon_0} = 1 + \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

- And we can now re-write the electric susceptibility in terms of the classical electron model and define the real and imaginary parts of χ as χ' and χ''

$$\chi = \chi' - i\chi'' = \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

Near Resonance Condition

- Near resonance, $\omega \approx \omega_0$, the second term in χ is small compared to the first term and the index of refraction can be approximated by $n=1+\chi/2$

$$n \approx 1 + \frac{\chi}{2} = 1 + \frac{Nq^2}{2m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

$$= 1 + \frac{Nq^2}{4\omega_0 m\varepsilon_0\left(\omega_0 - \omega + i\gamma/2\right)}$$

Complex Refractive Index and Dispersion

- The real and imaginary parts of the electric susceptibility and index of refraction give rise to ***optical phase delay*** and ***optical absorption***.
- We can separate the real and imaginary parts by multiplying numerator and denominator by the complex conjugate

$$\chi' - i\chi'' = \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)} \frac{\omega_0^2 - \omega^2 - i\gamma\omega}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$= \frac{Nq^2\left(\omega_0 - \omega\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]} - i\frac{Nq^2\left(\gamma/2\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

$$\chi' = \frac{Nq^2\left(\omega_0 - \omega\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

$$\chi'' = \frac{Nq^2\left(\gamma/2\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

Complex Refractive Index and Dispersion

- The complex index is (not assuming near resonance)

$$n' = n - i\kappa = 1 + \frac{Nq^2\left(\omega_0^2 - \omega^2\right)}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]} - i\frac{Nq^2\gamma\omega}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]}$$

- For a propagating field, we can write

$$E = Ae^{i(\omega t - k'z)} = Ae^{i(\omega t - \frac{2\pi}{\lambda}(n - i\kappa)z)}$$

$$= Ae^{i(\omega t - \frac{2\pi}{\lambda}nz)}e^{-\frac{2\pi}{\lambda}\kappa z}$$

- Where we can now define the attenuation coefficient α

$$\alpha = \frac{1}{I}\frac{dI}{dz}$$

$$I(z) = I(0)e^{-\alpha z}$$

$$\alpha = \frac{4\pi}{\lambda}\kappa$$

Complex Refractive Index and Dispersion

- And the change in refractive index as a function of frequency is defined as the ***Dispersion***.
- We can plot the real and imaginary parts as a function of frequency as absorption and phase curves